Dynamics of strongly tilted Hurricane Vortices

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Scaling Cascades in Complex Systems

Motivation

Structure of atmospheric vortices I: two scales

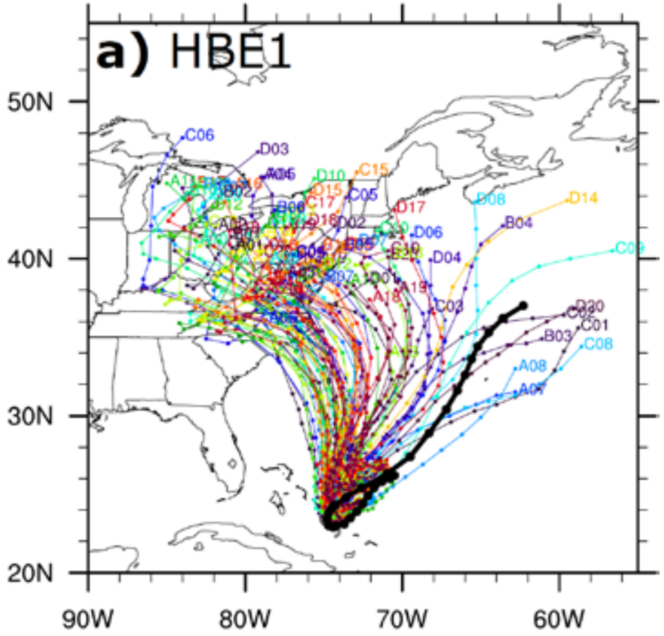
(Päschke et al., JFM, (2012))

Structure of atmospheric vortices II: cascade of scales

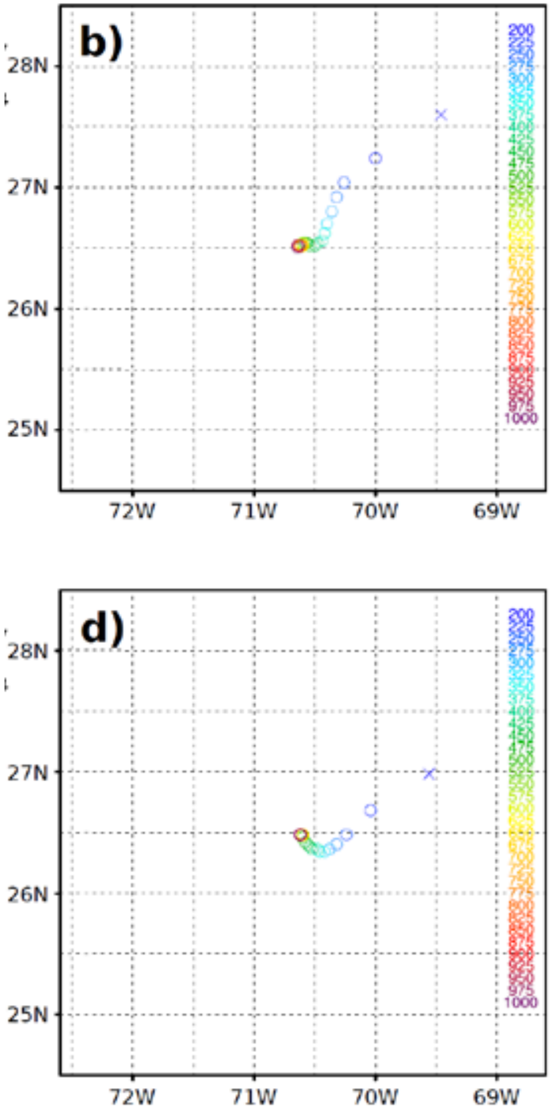
(Dörffel et al., arXiv:1708.07674)

Conclusions

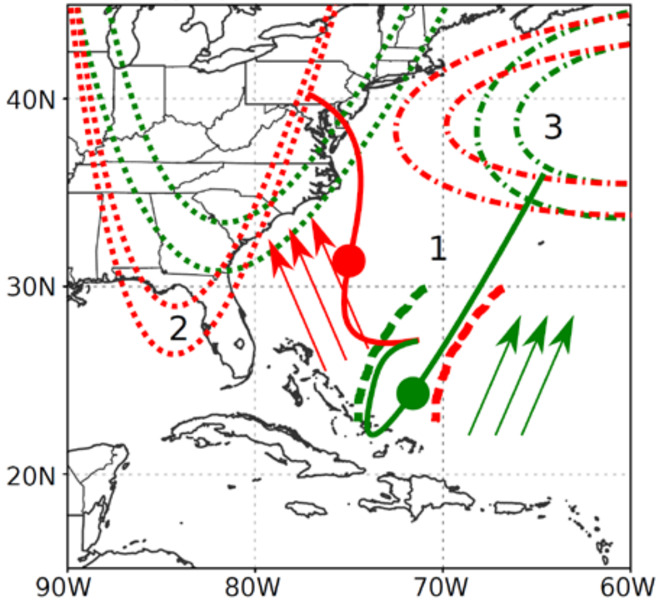
HWRF-Simulations of Storm "Joaquin"



Ensemble Tracks



Vortex Tilts



Storm Evolutions

Gh. Alaka et al. (2019), WAF, submitted

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Conclusions

Radial momentum balance regimes

$$- \frac{1}{\rho}\frac{\partial p}{\partial r} + f u_\theta = o(1) \Bigg($$	geostrophic	$\mathrm{Ro} \ll 1$	typical “weather”
$$\frac{u_\theta^2}{r} - \frac{1}{\rho}\frac{\partial p}{\partial r} + f u_\theta = o(1) \Bigg($$	**gradient wind**	$\mathrm{Ro} = \mathcal{O}(1)$	**tropical storm** **incipient hurricane**
$$\frac{u_\theta^2}{r} - \frac{1}{\rho}\frac{\partial p}{\partial r} = o(1) \Bigg($$	**cyclostrophic**	$\mathrm{Ro} \gg 1$	**hurricane**

Päschke, Marschalik, Owinoh, K., JFM, **701**, 137–170 (2012)

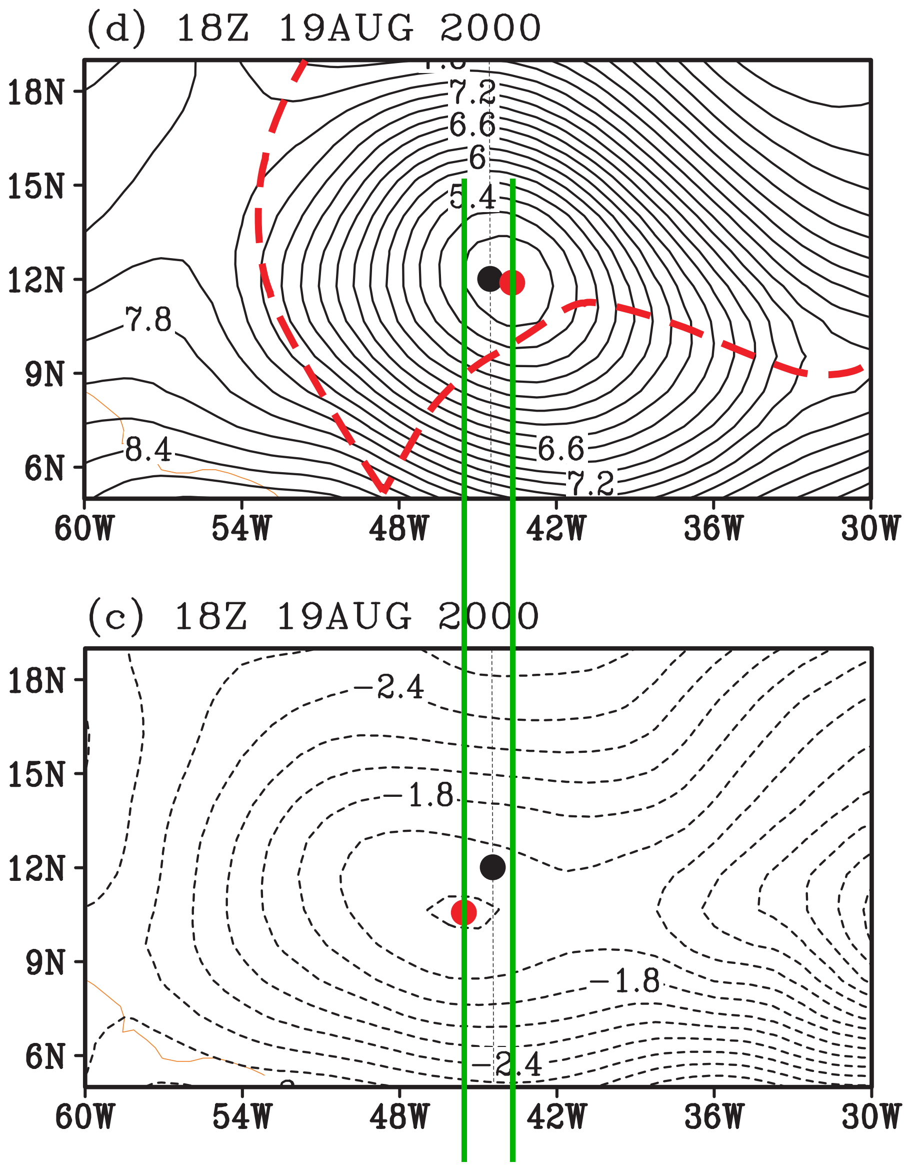
Dörffel et al., preprint, arXiv:1708.07674 (2017)

Vortex tilt in the incipient hurricane stage

(Velocity potential)

200 hPa
(∼ 12 km)

925 hPa
(∼ 0.8 km)



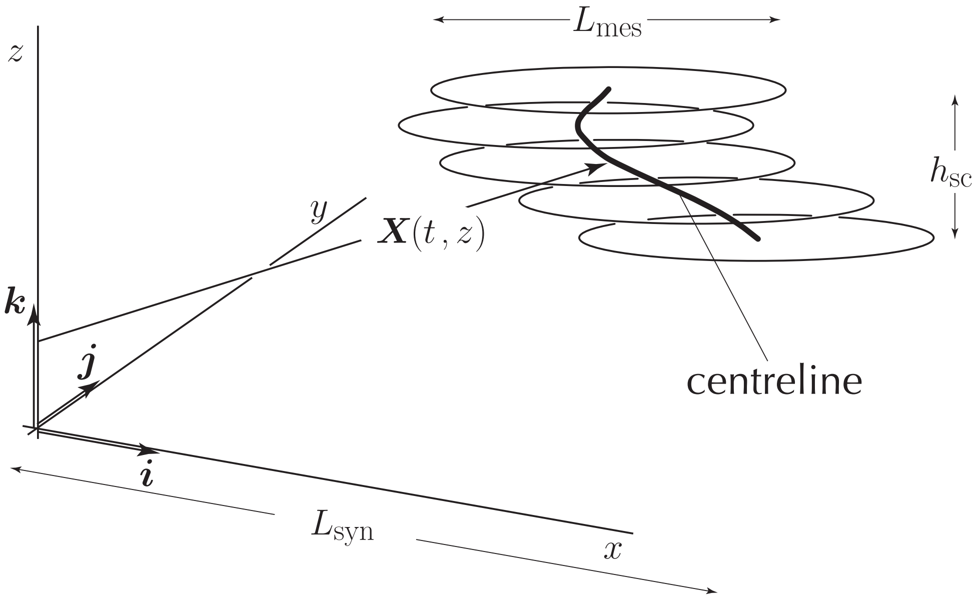
(d) 18Z 19AUG 2000

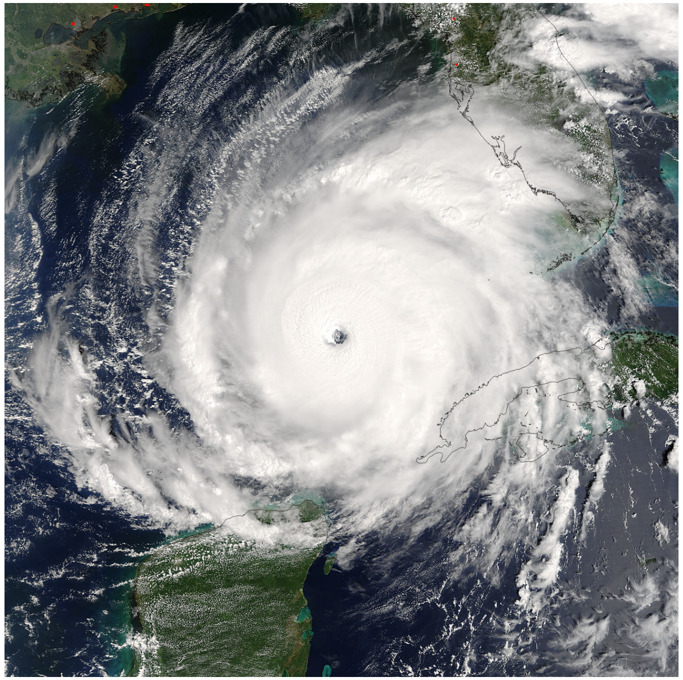
(c) 18Z 19AUG 2000

∼ **200 km**

Dunkerton et al., Atmos. Chem. Phys., **9**, 5587–5646 (2009)

Scaling regime





$$\underbrace{L_{\text{syn}}; \qquad |\boldsymbol{v}_{_{||}}| \sim v_{\text{syn}} \qquad t_{\text{syn}} \sim L_{\text{syn}}/v_{\text{syn}}}_{\text{farfield: classical QG theory}} \qquad \underbrace{L_{\text{mes}} = \sqrt{\varepsilon}L_{\text{syn}}; \qquad v_{\text{mes}} = \frac{1}{\sqrt{\varepsilon}}}_{\text{core: gradient wind scaling}}$$

$$\mathcal{C} \sim v_{\text{syn}}\,L_{\text{syn}}; \quad \text{Ro}_{\text{syn}} \sim \frac{v_{\text{syn}}}{f\,L_{\text{syn}}} = \mathcal{O}\left(\varepsilon\right) \qquad \mathcal{C} \sim v_{\text{mes}}\,L_{\text{mes}}; \quad \text{Ro}_{\text{mes}} \sim \frac{v_{\text{mes}}}{f\,L_{\text{mes}}} = \mathcal{O}\left(1\right)$$

Vortex motion ⇒ precessing quasi-modes*

Centerline evolution

$$\frac{\partial \widehat{\boldsymbol{X}}}{\partial \tau} = \underbrace{\widehat{\boldsymbol{X}} \cdot (\boldsymbol{\nabla}_{||} \overline{\boldsymbol{v}}_{\mathrm{QG}}) + \widehat{\boldsymbol{v}}^{*}_{\mathrm{QG}}}_{\text{background advection}} - \underbrace{\left(\ln \frac{1}{\sqrt{\varepsilon}} + \frac{1}{2}\right) (\boldsymbol{k} \times \chi)^{*} + (\boldsymbol{k} \times \boldsymbol{\Psi})}_{\text{self-induced motion}}$$

$$\chi = \text{fct}(\text{total circulation, centerline geometry})$$

$$\boldsymbol{\Psi} = \text{fct}(\textbf{core structure}\text{, centerline geometry, diabatic sources})$$

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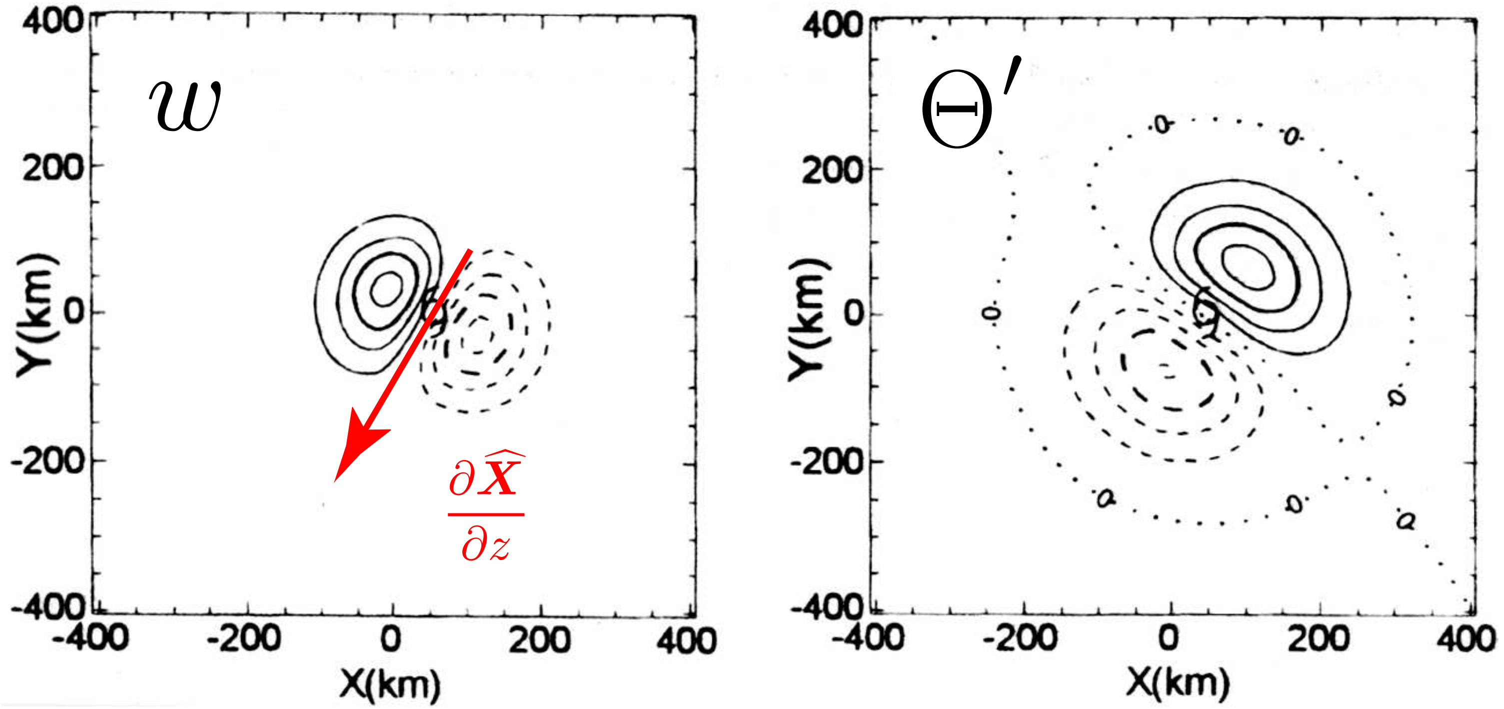
*effect of β-gyres; *akin to local-induction-approximation LIA *Grasso, Kallenbach, Montgomery, Reasor (1997, 2001, 2004)

The adiabatic lifting in a tilted vortex**



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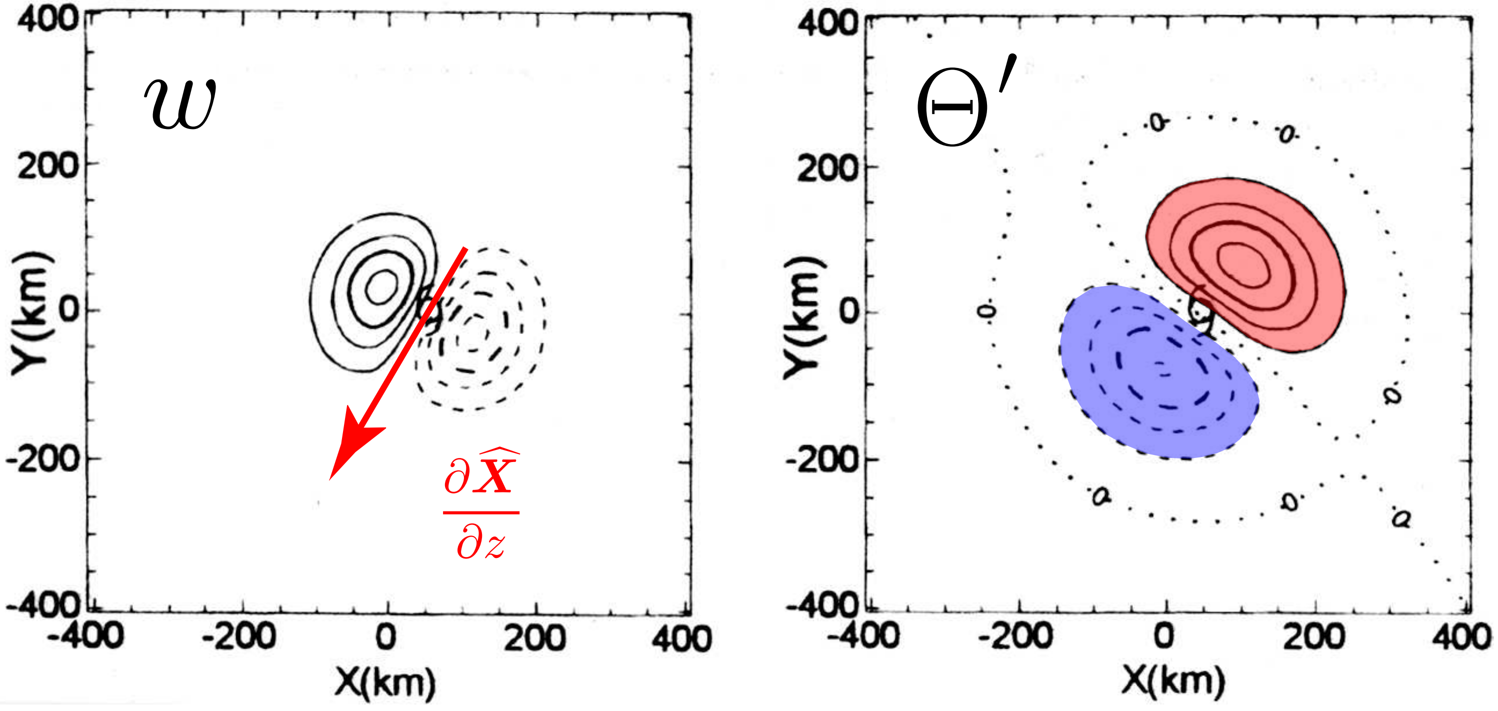
* Jones, Q.J.R. Met. Soc., **121**, 821–851 (1995)

* Frank & Ritchie, Mon. Wea. Rev., **127**, 2044–2061 (1999)

Heating pattern for max intensification (APE-theory)*

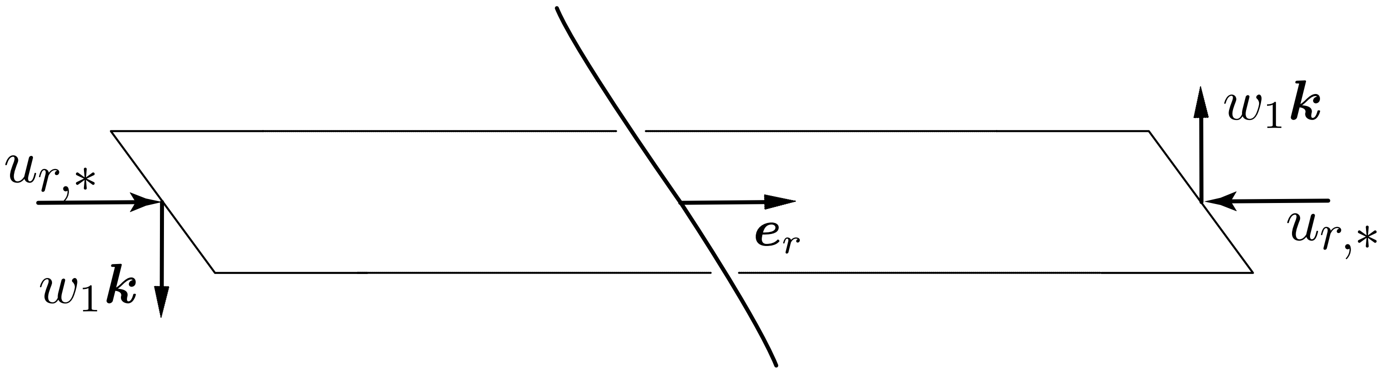


Lorenz, E. N., Generation of available potential energy and the intensity of the general circulation, *Tech. Rep.*, UCLA, (1955)

Radial transport by asymmetric heating

Circumferential Fouriermodes of vertical velocity

$$w_{1\boldsymbol{k}} = \frac{1}{d\overline{\Theta}/dz}\left[\underbrace{Q_{\Theta,1\boldsymbol{k}}}_{\text{WTG}} + \underbrace{\frac{\partial}{\partial z}\left(\boldsymbol{e}_r \cdot \widehat{\boldsymbol{X}}^{\perp}\right)_{\boldsymbol{k}} \left(\frac{u_\theta}{r}\left(\frac{u_\theta^2}{r} + f\,u_\theta\right)\right)}_{\text{adiabatic lifting}}\right]$$



$$\boldsymbol{u_{r,*}} = \left\langle w\,\frac{\partial}{\partial z}\left(\boldsymbol{e}_r \cdot \widehat{\boldsymbol{X}}\right)\right\rangle_\theta = \underline{\frac{1}{d\overline{\Theta}/dz}\left(Q_{\Theta,11}\frac{\partial \widehat{X}}{\partial z} + Q_{\Theta,12}\frac{\partial \widehat{Y}}{\partial z}\right)}$$

$(w = w_0 + w_{11}\cos\theta + w_{12}\sin\theta + \ldots)$

Spin-up by asymmetric heating

$(w = w_0 + w_{\mathbf{11}} \cos\theta + w_{\mathbf{12}} \sin\theta + \dots)$

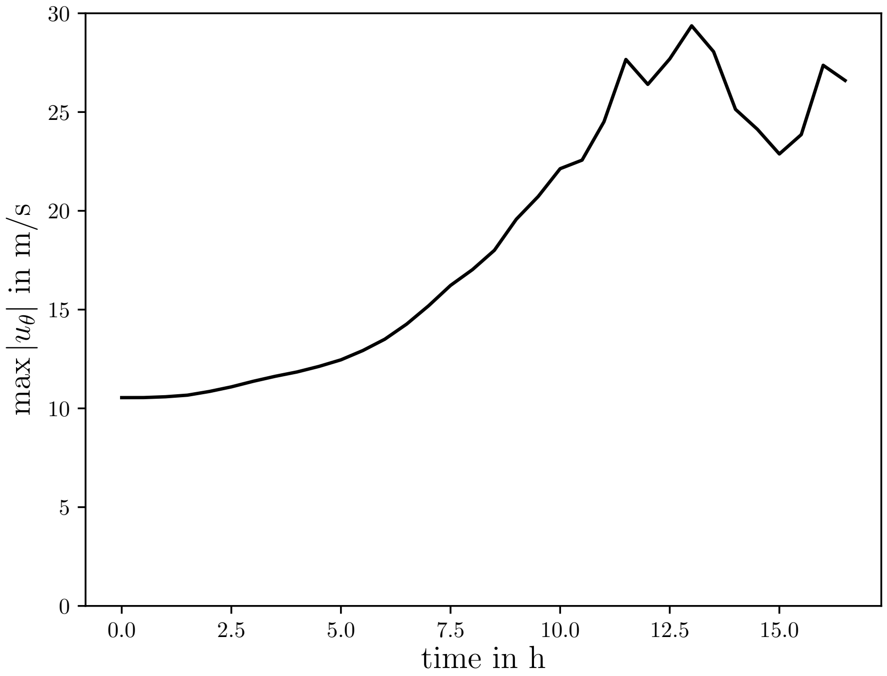
$$\underbrace{\frac{\partial u_{\theta,0}}{\partial \tau} + w_0 \frac{\partial u_{\theta,0}}{\partial z} + u_{r,00} \left(\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r} + f \right)}_{\Bigg(\text{standard axisym}\Bigg(\text{metric balance}} \Bigg(= - \boldsymbol{u_{r,*}} \left(\frac{u_\theta}{r} + f \right)$$

$$\boldsymbol{u_{r,*}} = \left\langle w \quad \boldsymbol{e}_r \cdot \frac{\partial \widehat{\boldsymbol{X}}}{\partial z} \Bigg) \right\rangle_\theta \Bigg(= \underline{\frac{1}{d\overline{\Theta}/dz} \quad Q_{\Theta,1\mathbf{1}} \frac{\partial \widehat{X}}{\partial z} + Q_{\Theta,1\mathbf{2}} \frac{\partial \widehat{Y}}{\partial z} \Bigg)} \Bigg($$

Päschke, Marschalik, Owinoh, K., JFM, **701**, 137–170 (2012)

Recent results

Qualitative corroboration through 3D-numerics



Artificial heating pattern:

$$w_{1\boldsymbol{k}} = \frac{1}{d\overline{\Theta}/dz}\Bigg[\Bigg(-\frac{\partial}{\partial z}\left(\boldsymbol{e}_r\cdot\widehat{\boldsymbol{X}}\right)_{\boldsymbol{k}}\Bigg(\frac{u_\theta}{r}\left(\frac{{u_\theta}^2}{r}+f\,u_\theta\right)\Bigg(+\frac{\partial}{\partial z}\left(\boldsymbol{e}_r\cdot\widehat{\boldsymbol{X}}^{\perp}\right)_{\boldsymbol{k}}\Bigg(\frac{u_\theta}{r}\left(\frac{u_\theta^2}{r}+f\,u_\theta\right)\Bigg]$$

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* Ultimately leaves asymptotic regime!

Dörffel et al., preprint, arXiv:1708.07674 (2017)

Recent results

Compatibility with Lorenz′ APE theory*

$$\Big(re_{\mathrm{k}}\Big)_t + \Big(ru_{r,0}[e_{\mathrm{k}}+p']\Big)_r + \Big(rw_0[e_{\mathrm{k}}+p']\Big)_z = \frac{r\overline{\rho}}{N^2\overline{\Theta}^2}\Big(\Bigg(\Theta'_0 Q_{\Theta,0} + \boldsymbol{\Theta}'_1 \cdot \boldsymbol{Q}_{\Theta,1}\Big)\Bigg($$

$$e_k = \frac{\overline{\rho}u_\theta^2}{2}$$

Symmetric & asymmetric are equally important !

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*Thanks to Olivier Pauluis!

Dörffel et al., preprint, arXiv:1708.07674 (2017)

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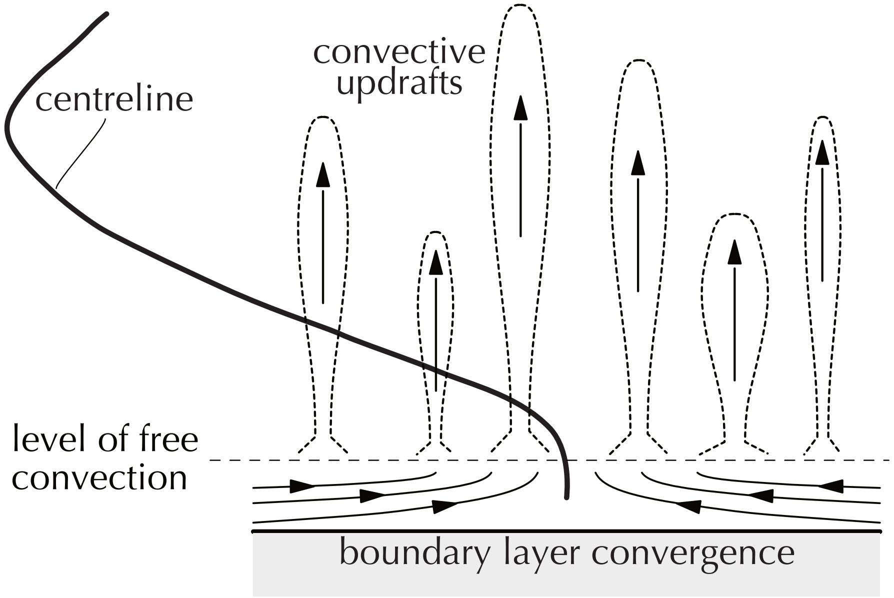
Convective updrafts



Convection concentrates in narrow towers $\left(\sqrt{\text{CAPE}} \sim 5\ldots30\,\text{m/s}\right)\Bigg($

Essentially dry dynamics between towers

Comparable average vertical mass fluxes

Spin-up by asymmetric convection

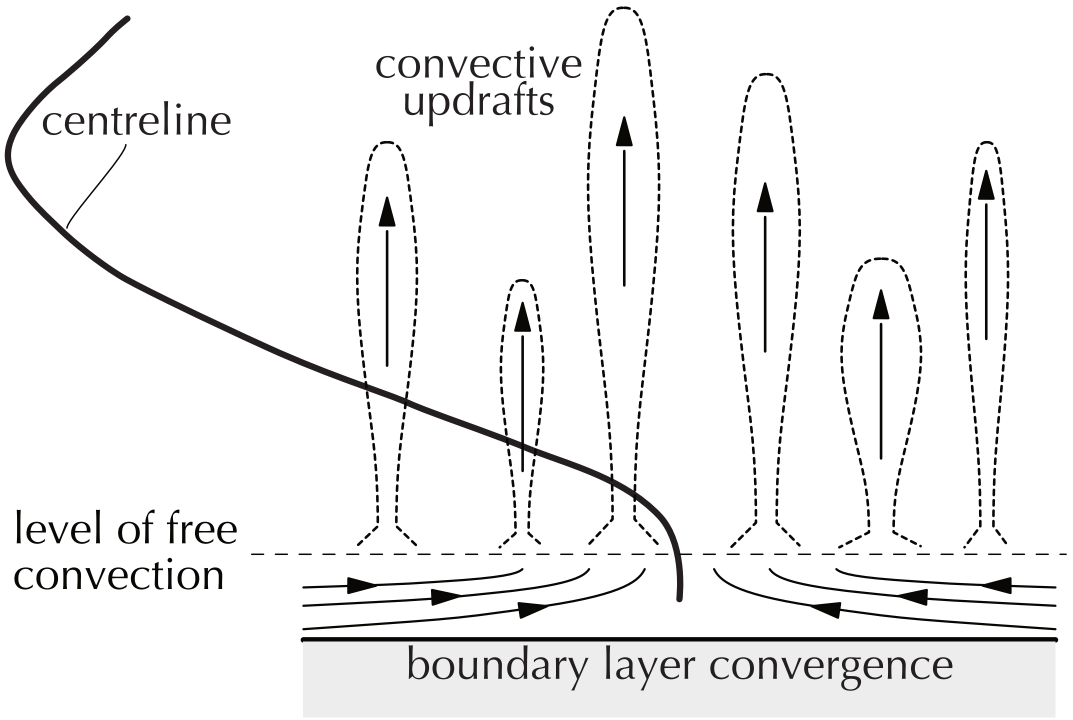
$$\underbrace{\frac{\partial u_{\theta,0}}{\partial \tau} + w_0 \frac{\partial u_{\theta,0}}{\partial z} + u_{r,00}\left(\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r} + f\right)}_{\text{standard axisymmetric balance}} = -\,\boldsymbol{u_{r,*}}\left(\frac{u_\theta}{r} + f\right)$$

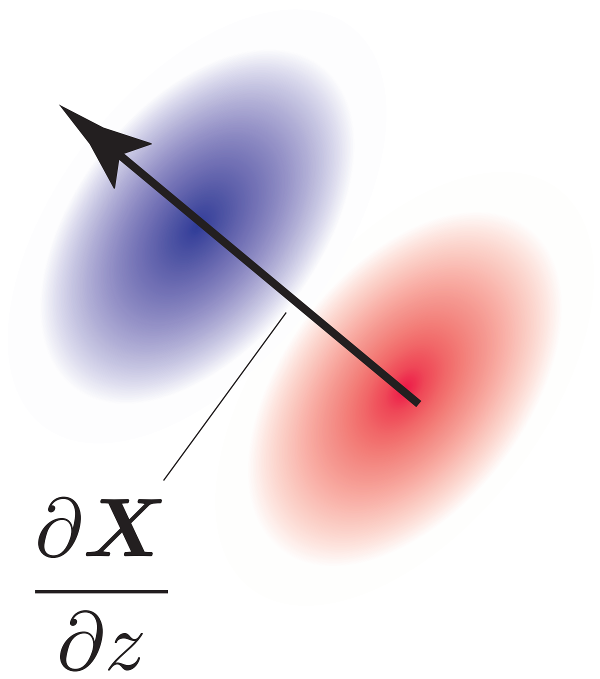
$$\boldsymbol{u_{r,*}} = \left\langle \boldsymbol{w}\,\frac{\partial}{\partial z}\left(\boldsymbol{e}_r \cdot \widehat{\boldsymbol{X}}\right)\right\rangle_\theta = \underline{\overline{\boldsymbol{w}}_{\mathrm{upd},1\mathbf{1}}\frac{\partial \widehat{X}}{\partial z} + \overline{\boldsymbol{w}}_{\mathrm{upd},1\mathbf{2}}\frac{\partial \widehat{Y}}{\partial z}}$$

!!

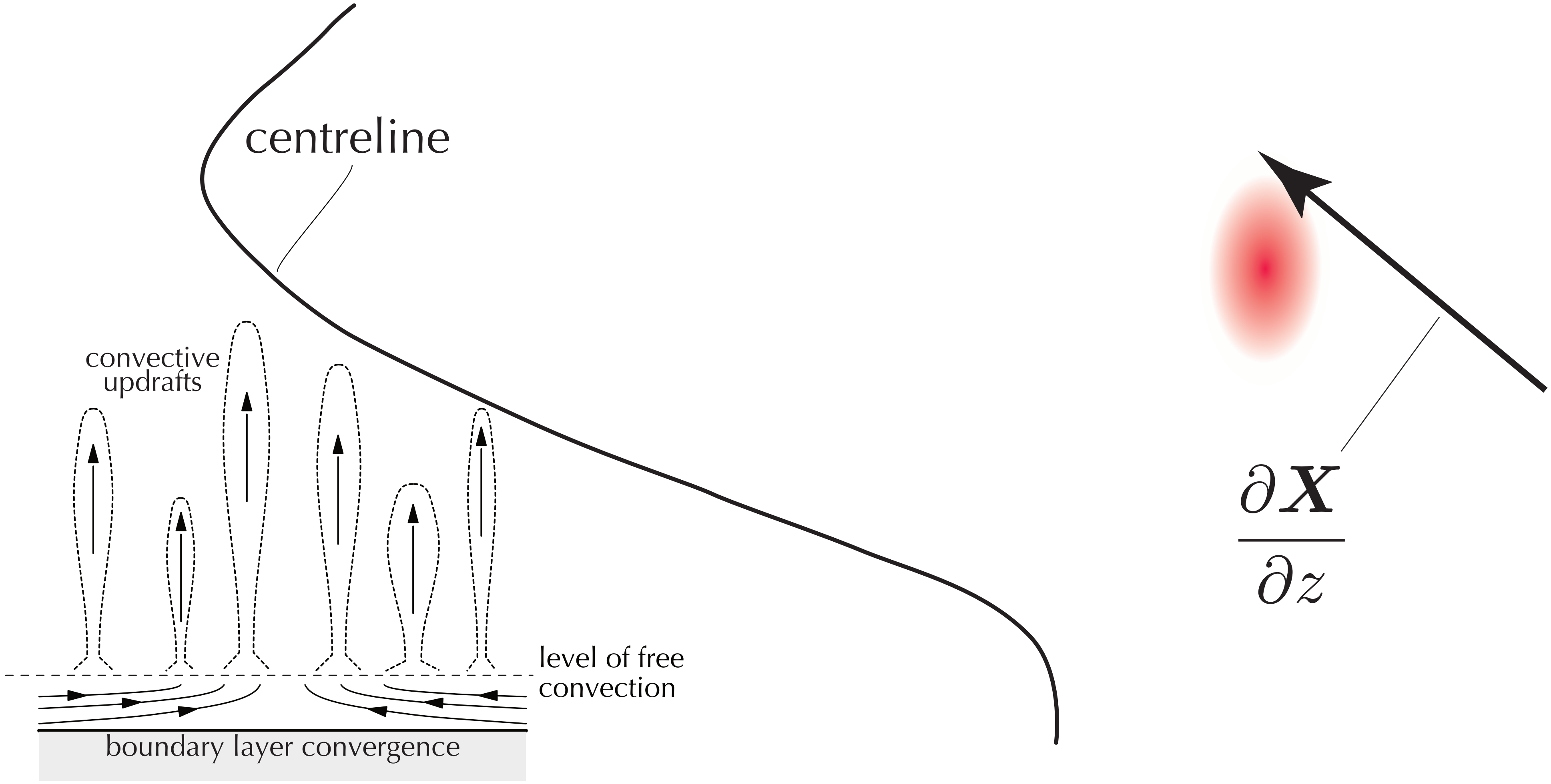
Area averaged updraft fluxes take role of heating-induced vertical velocities

Intensification & tilt destabilization





Attenuation / tilt stabilization



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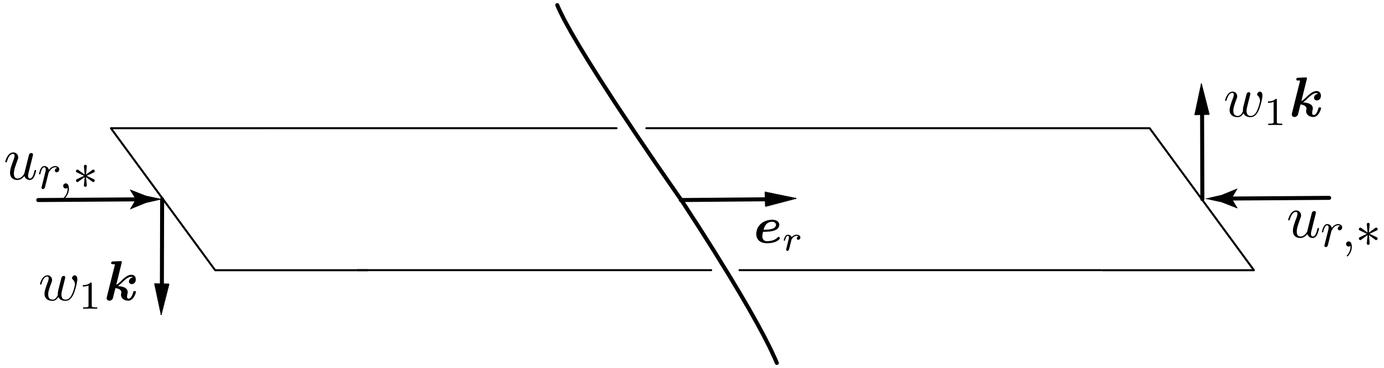
$$\boldsymbol{u_{r,*}} = \left\langle w \frac{\partial}{\partial z}\left(\boldsymbol{e}_r \cdot \widehat{\boldsymbol{X}}\right)\right\rangle_\theta \Bigg(= \underline{\frac{1}{d\overline{\Theta}/dz} \quad Q_{\Theta,1\mathbf{1}} \frac{\partial \widehat{X}}{\partial z} + Q_{\Theta,1\mathbf{2}} \frac{\partial \widehat{Y}}{\partial z}\Bigg)} \Bigg($$



Radial transport in a tilted vortex induced by asymmetric heating